AN ELECTROMAGNET WITH HALL EXCITATION

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UDC 538.4+538.311

In 1911 Corbino showed that if a disk with a current flowing to the axis is placed in a magnetic field parallel to the axis of the disk, then due to the Hall emf the initially straight line of electric current is turned into a spiral. This leads to an increase in the length of the current line and thus to an increase in the disk resistance. The change in the disk resistance in a magnetic field was used in [1] to switch the current in the circuit of an inductive energy store. If the electric current carriers move from the edge of the disk to the axis, the azimuthal Hall current is accompanied by an increase in the magnetic field inside the disk compared with that outside it [2]. The same processes occur in a hydromagnet [3-5], in which the radial flow of a conducting liquid in an axial magnetic field is used to amplify a magnetic field. In the papers mentioned earlier the transients which occur when the steady magnetic field is established were not considered. To produce a magnetic field, and particularly for switching, the switch-on time of the device is of considerable importance. Hence, in this paper we consider the nonstationary problem of the amplification of a magnetic field. The amplification of the field is obtained and the time taken for the stationary state to build up is found. Both quantities depend exponentially on the magnetic Reynolds number. For a hydromagnet it is shown that the steady-state magnetic field differs considerably from that obtained in [4, 5]. The disagreement between the results is due to the fact that the boundary conditions in [4, 5] were arbitrarily chosen.

§1. We will consider a number of problems connected with the excitation of a Hall current which amplifies the magnetic field applied to the device. Figure 1 shows an arrangement which leads to amplification of the magnetic field in a plane geometry. The current I_0 flows in the conducting medium from the plane C_1 to A_1 and from C_2 to A_2. The magnetic field is perpendicular to the plane of Fig. 1. The Hall current which occurs in the left part of the arrangement is closed by the same Hall current on the right. A sketch of the axisymmetrical case is shown in Fig. 2. Here the current I_0 is radial, while the Hall current is azimuthal. The problem reduces to solving the following set of equations:

$$\operatorname{rot} \mathbf{B} = \mu_0 \mathbf{j},\ \operatorname{rot} \mathbf{E} = -\partial \mathbf{B}/\partial t,\ \mathbf{j} = \sigma(\mathbf{E} + (1/en)[\mathbf{j} \times \mathbf{B}]),\quad \operatorname{div} \mathbf{j} = 0, \tag{1.1}$$

where we have used the usual notation for the quantities.

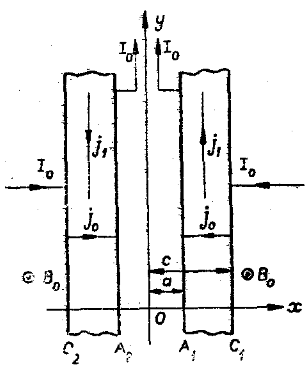

Fig. 1

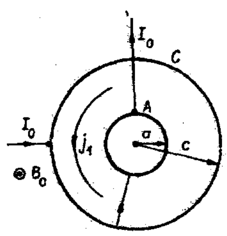

Fig. 2

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 44-48, May-June, 1979. Original article submitted February 9, 1978.

0021-8944/79/2003-0294$07.50

The following initial and boundary conditions are of physical interest:

1. The circuit is connected to an external dc source so that the current flows from the outer boundary C to the inner boundary A. Initially the field B_z is everywhere zero. At the instant $t=0$ a field $B_z|_C = B_0$ is switched on on the outer boundary and then remains constant. On the boundary C the field is continuous. The field in the cavity is uniform and depends only on time. The boundary condition on the inner surface A follows from the first, second, and third equations of system (1.1):

$$\mathrm{rot}\,\mathbf{B}|_A = \mu_0\sigma(\mathbf{E} + (1/en)[\mathbf{j} \times \mathbf{B}])|_A. \tag{1.2}$$

This problem describes the operation of a current switch in the circuit of an inductive energy store. It can be used to control a constant external magnetic field.

2. The field at the initial instant of time is everywhere equal to $B_z = B_0$. At the instant of time $t=0$ the current I_0 is switched on and then remains constant. The boundary condition on the inner surface is (1.2) as before. This formulation of the problem describes the operation of a system for amplifying a magnetic field due to the operation of an external dc source.

3. Since it was assumed in [4, 5] that the field is zero on the outer boundary, we will specially consider problem 2 with the condition $B_z|_C = 0$.

Henceforth the interaction between the current I_0 and its own magnetic field will be ignored, since this interaction, when the current conducting leads are appropriately placed, leads merely to polarization of the conducting medium along the z axis.

§2. Solutions of the above problems can be obtained by means of the Laplace transform [6]. Thus, problem 1 in the plane case has the solution

$$B_z = B_0 e^{2\alpha(c-x)} \times \left[1 - e^{-\alpha(c-x)} \sum_{i=1}^{\infty} e^{-\frac{\alpha^2 - p_i}{\mu_0\sigma}t} \frac{2\sqrt{p_i}}{\alpha^2 - p_i} \frac{\mathrm{sh}\sqrt{p_i}\,(c-x)}{\dfrac{\alpha(1+\alpha a) + a p_i}{[\alpha(1+\alpha a) - a p_i]^2 - p_i} - (c-a)}\right], \tag{2.1}$$

where p_i are the consecutive nontrivial roots of the equation

$$\mathrm{th}\sqrt{p}(c-a) = \sqrt{p}/[\alpha(1+\alpha a) - ap]. \tag{2.2}$$

The combination $2\alpha(c-a) = (c-a)\mu_0\sigma j_0/en$, where j_0 is the current density flowing from C to A, is the ratio of the field diffusion time into the thickness $(c-a)$ to the time taken for the carriers to move from C to A, i.e., the magnetic Reynolds number Re_m.

For the axisymmetrical case the solution of problem 1 has the form

$$B_z = B_0\left(\frac{c}{r}\right)^{2\nu}\left[1 + \left(\frac{r}{c}\right)^{\nu}\frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} e^{\frac{pt}{\mu_0\sigma}}\frac{dp}{p}\right.$$

$$\times \frac{\left[K_{\nu-1}(a\sqrt{p}) + \frac{a\sqrt{p}}{2}K_\nu(a\sqrt{p})\right]I_\nu(r\sqrt{p})}{\left[K_{\nu-1}(a\sqrt{p}) + \frac{a\sqrt{p}}{2}K_\nu(a\sqrt{p})\right]I_\nu(c\sqrt{p})} \rightarrow$$

$$\left.\rightarrow \frac{+\left[I_{\nu-1}(a\sqrt{p}) - \frac{a\sqrt{p}}{2}I_\nu(a\sqrt{p})\right]K_\nu(r\sqrt{p})}{+\left[I_{\nu-1}(a\sqrt{p}) - \frac{a\sqrt{p}}{2}I_\nu(a\sqrt{p})\right]K_\nu(c\sqrt{p})}\right], \tag{2.3}$$

where I_ν, $I_{\nu-1}$, and K_ν, $K_{\nu-1}$ are Bessel functions of imaginary argument; the quantity $2\nu = \mu_0\sigma j_0 c/en = \mathrm{Re}_m$ as before is the magnetic Reynolds number, and j_0 is the radial current density on the outer boundary.

The solutions of problem 2 have a similar structure.

To analyze the steady state of a hydromagnet we will solve problem 3 in the plane case. For clarity we will assume $a=0$, or, which is the same thing, that the boundary A is impenetrable for the magnetic field:

$$B_z = B_0 e^{\alpha(c-x)}\sum_{i=1}^{\infty} e^{-\frac{\alpha^2 - p_i}{\mu_0\sigma}t}\frac{\sqrt{p_i}\,\mathrm{sh}\sqrt{p_i}(c-x)}{\alpha - c(\alpha^2 - p_i)}\left[1 - \frac{2\alpha}{\sqrt{\alpha^2 - p_i}}e^{-\alpha c}\right], \tag{2.4}$$

where p_i are the roots of Eq. (2.2) for $a = 0$. The solution of the axisymmetrical problem 3 will not be given here because of its complexity.

§3. The structure of the solutions obtained for problems 1 and 2 have the same form: Each of them contains a series which decays exponentially with time, and a term independent of the time. The solution of problem 3 does not have such a term.

Analysis of the solutions for small magnetic Reynolds numbers leads to the conclusion that the fields, and the times taken for them to become established, are practically unchanged compared with the usual problems of the diffusion of a field into a conductor.

The situation changes considerably for large magnetic Reynolds numbers ($\mathrm{Re}_m \gg 1$). In this case Eq. (2.2) has an infinite set of negative roots $p_i < 0$ and one root $p \sim \alpha^2$. The terms of series (2.1) when $p_i < 0$ differ slightly from the corresponding terms of the series in the usual problem of the diffusion of a field ignoring the Hall effect, so that they disappear in a time on the order of the time of free diffusion of the field into the specimen. Using this we can represent solution (2.1) for fairly long times in the form

$$B_z \approx B_0 e^{\mathrm{Re}_m\left(1-\frac{x}{c}\right)}\left[1 - e^{-\frac{t}{\tau}}\right],$$

where

$$\tau \approx \frac{\mu_0 \sigma c^2}{\mathrm{Re}_m^2} e^{\mathrm{Re}_m} \tag{3.1}$$

is the characteristic field buildup time. For simplicity, we have here used the approximation $a/c \ll 1$. This time also represents the buildup of the field in problems 2 and 3 for plane geometry.

A similar situation occurs in the axisymmetrical problem. The expression under the integral in (2.3) has a singular point $|p| \to 0$ when $\mathrm{Re}_m \to \infty$, which also determines the rate of variation of the field inside the device. Using the asymptotic expansion of the Bessel functions for large values of the index, we can obtain from (2.3)

$$B_z \approx B_0 \left(\frac{c}{r}\right)^{\mathrm{Re}_m}\left[1 - e^{-\frac{t}{\tau}}\right],$$

where

$$\tau \approx \frac{\mu_0 \sigma c^2}{\mathrm{Re}_m^2}\left(\frac{c}{a}\right)^{\mathrm{Re}_m}. \tag{3.2}$$

The stationary distribution of the field between the cylinders A and C in problems 1 and 2 has the form

$$B_z = B_0 \left(\frac{c}{r}\right)^{\mathrm{Re}_m} \tag{3.3}$$

for any magnetic Reynolds numbers.

The steady-state value of the magnetic field in problem 3 is zero, which follows from (2.4), while the relaxation time when $\mathrm{Re}_m \gg 1$ is determined by relations (3.1) or (3.2) depending on the geometry.

Hence, we have shown that amplification of the magnetic field occurs if the field on the outer boundary differs from zero. When the field on the outer boundary is zero no amplification occurs, and the relaxation time is given by relations (3.1) or (3.2) for $\mathrm{Re}_m \gg 1$.

§4. The solutions of the problems obtained in the previous sections can be applied to analyze the transient in a hydromagnet if we make the substitution $j_0/en = u_0$ in the magnetic Reynolds number, where u_0 is the velocity of the normal flow of liquid on the outer boundary.

Thus, in the axisymmetrical problem, if the field on the outer boundary differs from zero, the field distribution in the hydromagnet is given by relation (3.3). If the field on the outer boundary is equal to zero, the steady-state value of the field inside the hydromagnet will be zero, which differs from the results obtained in [4, 5]. The time taken for the hydromagnet to reach the steady state is given by the same relations as in the Corbino disk device.

§5. In addition to problems 1-3 we will consider the problem of the concentration of the magnetic field when the flux through the hydromagnet remains constant. This condition was used in [4, 5].

The outer boundary of the hydromagnet is made of a good conductor, so that the diffusion of the field through it during the contraction period can be neglected. The field in the hydromagnet before contraction is $B_z = B_0$. Using the condition for the conservation of the flux and the field distribution (3.3) we obtain

$$B_z = B_0\left(\frac{c}{r}\right)^{\mathrm{Re}_m} \quad \frac{(\mathrm{Re}_m - 2)a^{\mathrm{Re}_m - 2}}{\mathrm{Re}_m\, c^{\mathrm{Re}_m - 2} - 2a^{\mathrm{Re}_m - 2}}.$$

It can be seen from this equation that the maximum achievable field in the center of the hydromagnet under flux-conservation conditions is $B_z = B_0(c/a)^2$ as $\mathrm{Re}_m \to \infty$. If the field on the boundary is kept constant, the field at the center of the hydromagnet is $B_z = B_0(c/a)^{\mathrm{Re}_m}$; i.e., more effective amplification of the field occurs.

The author thanks E. I. Bichenkov and R. L. Rabinovich for useful discussions and advice.

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ELECTRIC FIELDS IN A SINGLE-TURN MAGNETIC GENERATOR WITH A PARABOLIC TURN PROFILE

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UDC 538.4

1. To solve a number of problems in experimental physics, it is necessary to have available a wide range of high-power electromagnetic energy [1]. One of the possible pulsed sources of electromagnetic energy with high specific energy capacity and power is the explosive-driven magnetic generator. Several types of explosive-driven magnetic generators are known at the present time [2-5]. The variety of generators is due to the need to satisfy contradictory requirements (e.g., such as a short operating time and a large initial inductance, a large value of the generated current, and limited dimensions of the current circuit), which are difficult to combine in a single generator.

In any type of explosive-driven magnetic generator when the condition $dL(t)/dt \gg R(t)$ is satisfied, the main increase in the generated current occurs at the end of the deformation of the electric circuit, $I(t) \sim L^{-1}(t) \cdot \eta(t)$ (I, L, and R are the current, inductance, and total resistance of the generator). The efficiency of the operation of the generator, or the value of the magnetic flux conservation coefficient η, at this stage of the magnetic-cumulative process may be reduced due to electrical breakdowns occurring in the air which fills the compression volume of the generator. The breakdown mechanism, accompanied by the cutting of part of the inductance of the circuit, and the related loss in magnetic flux, leads to considerable limitations of the electromagnet energy ($W \sim \eta^2$), and also to a reduction in the current gain ($k_T \sim \eta$), and the energy gain ($k_e \sim \eta^2$), and the fraction of the lost flux increases toward the end of the operation of the electromagnet. Increasing the coefficient η to the level specified by diffusion of the magnetic field during the compression and displacement of the flux into the generator load, is one of the main problems in constructing small-size explosive-driven magnetic generators with high output characteristics, since the technological factors (e.g., for spiral explosive-driven magnetic generators, the decentering of the spiral coil from the central tube), which affect the coefficient η, may be eliminated or reduced to a minimum. Thus, in [6] certain recommendations are made, confirmed experimentally, to achieve this purpose.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 49-58, May-June, 1979. Original article submitted June 15, 1978.